Northwestern University EECS 203
Midterm Solutions for Problems #1,2

1)
In order to give the minimal POS form for the following function, we start by drawing the K-Map, and then circle the zeros in order to determine $\overline{f}$.

$$f(a,b,c,d) = \sum(0,2,3,4,12,14) + d(7,8)$$

ab\cd	00	01	11	10
00	1	0	1	1
01	1	0	X	0
11	1	0	0	1
10	X	0	0	0

We can now use DeMorgan's laws to solve for *f*.

$$\overline{f} = \bar{c}d + ad + a\bar{b} + \bar{a}bc$$

$$f = \overline{\bar{c}d + ad + a\bar{b} + \bar{a}bc}$$

$$f = (\overline{\bar{c}d})(\overline{ad})(\overline{a\bar{b}})(\overline{\bar{a}bc})$$

$$\boxed{f = (c+\bar{d})(\bar{a}+\bar{d})(\bar{a}+b)(a+\bar{b}+\bar{c})}$$

2)
Solved using Quine-McCluskey Method:

$$f = \bar{a}\bar{b} + \bar{b}c + a\bar{b}\bar{c} + \bar{a}b\bar{c}$$

Truth Table:

a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Quine-McCluskey Chart:

# of 1's			
0	0 000	0,1 00X 0,2 0X0 0,4 X00	0,1,4,5 X0X
1	1 001 2 010 4 100	1,5 X01 4,5 10X	
2	5 101		

Quine-McCluskey Table:

Minterm	0X0	X0X
0	√	√
1		√
2	√	
4		√
5		√

Based on the tables, we get the solution, $\boxed{f = \bar{b} + \bar{a}\bar{c}}$

2)

Solved using algebraic manipulation:

Manipulation	Reason
$f = \bar{a}\bar{b} + \bar{b}c + a\bar{b}\bar{c} + \bar{a}b\bar{c}$	Given
$f = \bar{b}(\bar{a} + c + a\bar{c}) + \bar{a}b\bar{c}$	$x(y+z) = xy+xz$
$f = \bar{b}(\bar{a} + a\bar{c} + c + a\bar{c}) + \bar{a}b\bar{c}$	$x = x+x$
$f = \bar{b}((\bar{a} + a)(\bar{a} + \bar{c}) + (c + a)(c + \bar{c})) + \bar{a}b\bar{c}$	$x+yz = (x+y)(x+z)$
$f = \bar{b}(\bar{a} + \bar{c} + c + a) + \bar{a}b\bar{c}$	$x + \bar{x} = 1$
$f = \bar{b} + \bar{a}b\bar{c}$	$x + \bar{x} = 1$
$f = (\bar{b} + b)(\bar{b} + \bar{a}\bar{c})$	$x+yz = (x+y)(x+z)$
$f = \bar{b} + \bar{a}\bar{c}$	$x + \bar{x} = 1$, $x \cdot 1 = x$

Hence the solution is $\boxed{f = \bar{b} + \bar{a}\bar{c}}$

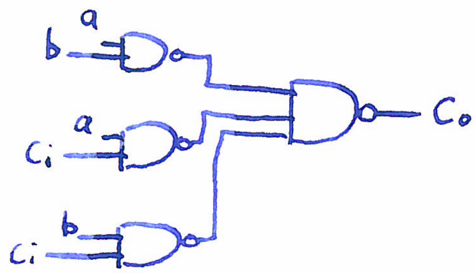
3) $C_o(a, b, c_i) = ab + ac_i + bc_i$

$$= \overline{\overline{ab + ac_i + bc_i}}$$

$$= \overline{\overline{ab} \cdot \overline{ac_i} \cdot \overline{bc_i}}$$

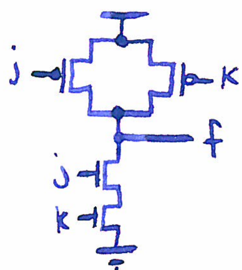
C_o	ab			
c_i	0	0	1	0
	0	1	1	1

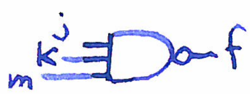
Already minimal.



This is fine, as is a single-gate design.

$\equiv$





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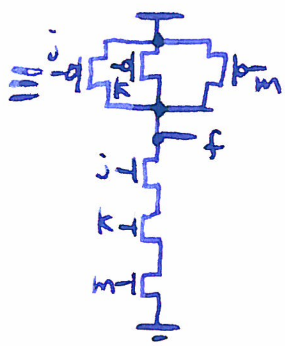


4) Valid input combinations:

a	b	c
0	1	1
1	0	1
1	1	0

Three combinations.

$$\lceil \lg_2 3 \rceil = 2$$

Two output bits required (p, q).

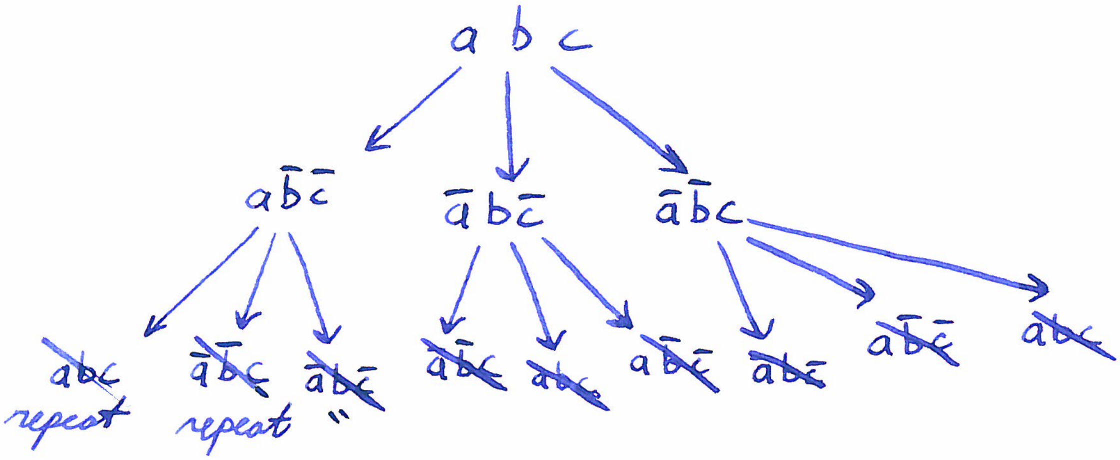
a	b	c	p	q
0	1	1	0	1
1	0	1	1	0
1	1	0	1	1

If we used 00, 01, 10, we would have a valid but more costly implementation.

Instead, just pick any two inputs allowing the input combination to be uniquely identified.

$$\boxed{\begin{aligned} p &= a \\ q &= b \end{aligned}}$$

5) We will determine the set of all 3-bit numbers that may be reached by taking 2-bit steps from some initial number, abc.



Thus, exactly four numbers can be reached by changing two bits at a time.

$4 < 8$

No such encoding is possible. □

A similar Karnaugh map based proof is also fine.

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6) The charge carriers in PMOS transistors (holes) move approximately half as fast as the charge carriers in NMOS transistors (electrons). Doubling the width compensates for this, balancing speed.

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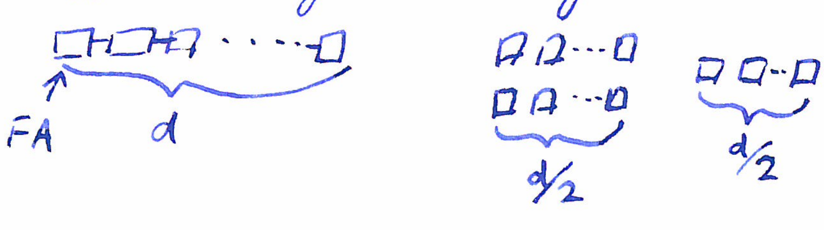
7) Ripple carry — Carry select



$\boxed{d/2}$