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Definition: Unate covering

Given a matrix for which all entries are 0 or 1, find the minimum cardinality subset of columns such that, for every row, at least one column in the subset contains a 1.

I'll give an example

Cyclic core

		bc			
		00	01	11	10
a	0	0	1	1	1
	1	1	1	0	1

  

	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Eliminate rows covered by essential columns

	A	B	C
H		1	
I	1		1
J	1	1	
K		1	1

Review: Quine–McCluskey two-level logic minimization

- Compute prime implicants with a well-defined algorithm
  - Start from minterms
  - Merge adjacent implicants until further merging impossible
- Select minimal cover from prime implicants
  - Unate covering problem
- What is happening?
  - $ab + a\bar{b} = a$

Prime implicant selection

Prime implicants  
Use these to...

	01X	0X0	X00	X11
<del>000</del>		1	1	
<del>010</del>	1	1		
<del>011</del>	1			1
<del>111</del>				1
<del>100</del>			1	

On-set minterms  
... cover these

Implicant selection reduction

- Eliminate rows covered by essential columns
- Eliminate rows dominated by other rows
- Eliminate columns dominated by other columns

Eliminate rows dominated by other rows

	A	B	C
H	1		
I	1	1	
J	1		1

## Eliminate columns dominated by other columns

	A	B	C
H	1		
I	1	1	
J	1		1
K		1	

## Use bound to constrain search space

- Eliminate rows covered by essential columns
- Eliminate rows dominated by other rows
- Eliminate columns dominated by other columns
- Speed improved, still  $\in \mathcal{NP}$ -complete
  - Too slow to solve for large problem instances

## Another example

$$f(a, b, c) = \sum(1, 2, 6) + d(3)$$

## Reading assignment

- M. Morris Mano and Charles R. Kime. *Logic and Computer Design Fundamentals*. Prentice-Hall, NJ, fourth edition, 2008
- Rest of Section 4.6

## Backtracking

- Will proceed to complete solution unless cyclic
- If cyclic, backtrack
  - Try all possible options to completion
- Advanced topic: Can use a number of tricks to simplify this

## Loose end – Don't cares

- What should be done about Xs in QM?
- Should they be included in the initial minterms?
- Should they be required in the Unate Covering problem?

## Summary

- Review
- Prime implicant selection in Quine–McCluskey
- Encoders and decoders
- Review: Transmission gates
- Multiplexers and demultiplexers

## Computer geek culture reference

- Complexity classes
- Michael R. Garey and David S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman & Company, NY, 1979