(1) 1)a) $f(a,b,c,d) = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}c\bar{d} + \bar{a}b\bar{c}d + \bar{a}bcd + a\bar{b}c\bar{d} + ab\bar{c}\bar{d} + ab\bar{c}d + abcd$

36 literals

(2) 1)b) Literal count is correlated with area.

(7) 1)c)

0000✓	000X	X1X1
0001✓	00X0	
0010✓	0X01	
0101✓	X010	
1010✓	01X1✓	
1100✓	X101✓	
0111✓	110X	
1101✓	X111✓	
1111✓	11X1✓	

	000X	00X0	0X01	X010	110X	X1X1
0000	1	1				
0001	1		1			
0010		1		1		
0101			1			1
1010				1		
1100					1	
0111						1
1101					1	1
1111						1

$$f(a,b,c,d) = \bar{a}\bar{b}\bar{c} + \bar{b}c\bar{d} + ab\bar{c} + bd$$

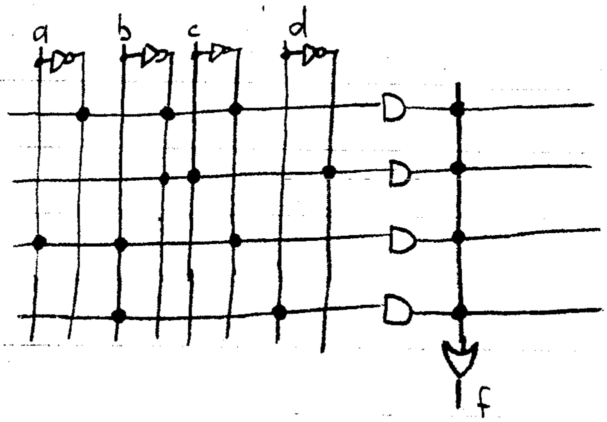
1 1)d) 11 literals

2 1)e) Yes, for SOP.

2 1)f) Unate covering

2 1)g) Worst-case time is exponential in input size.

③ 1)h)



⑧ 2)a) $\bar{a}\bar{b}\bar{c} + \bar{b}c\bar{d} + ab\bar{c} + bd$

cube	prim. div.	kernel label
$\bar{b}$	$\bar{a}\bar{c} + c\bar{d}$	k_1
b	$a\bar{c} + d$	k_2
$\bar{c}$	$\bar{a}\bar{b} + ab$	k_3

Using k_1: $(\bar{a}\bar{c} + c\bar{d})\bar{b} + ab\bar{c} + bd$

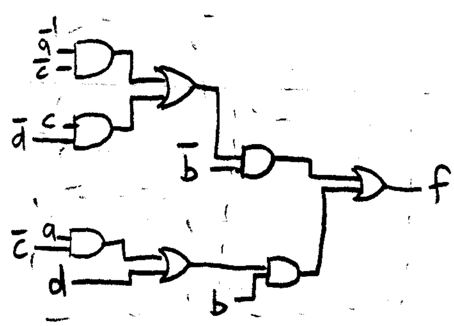
cube	prim. div	kernel label
b	$a\bar{c} + d$	k_4

Using k_4: $(\bar{a}\bar{c} + c\bar{d})\bar{b} + (a\bar{c} + d)b$

① 2)b) 9 literals

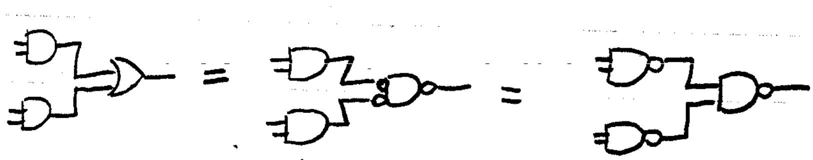
(2) 2)c) No. We used a potentially sub-optimal heuristic

(3) 2)d)

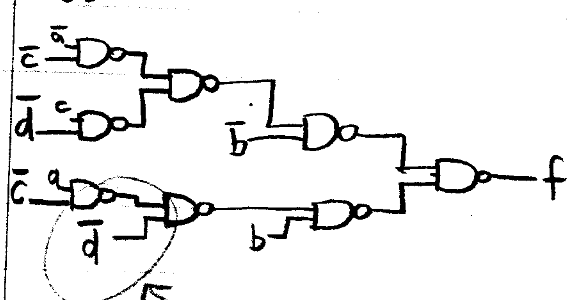


(1) 2)e) 4 levels

(3) 2)f)



so...



need to complement d

(2) 2)g) Some CLBs can implement arbitrary functions of 4 variables: 1 CLB required.

③ 3)a) It eliminates redundant cubes from the cover.

④ 3)b) Remove the cube from the cover, cofactor by the cube, if the resulting cover is not tautological, the cube was relatively essential.

⑤ 3)c)

XX00
0XXX
X1X1
1X0X
XX11

3)c)

$$\begin{pmatrix} \cancel{xx00} \\ 0xxx \\ 1x0x \\ xx11 \end{pmatrix}_{x1x1} = \begin{matrix} 0xxx \\ 1x0x \\ xx1x \end{matrix}$$

$$\swarrow^{\bar{a}} \quad \searrow^{a}$$

$$\boxed{\begin{matrix} xxxx \\ xx1x \end{matrix}} \qquad \begin{matrix} xx0x \\ xx1x \end{matrix}$$

taut.

$$\swarrow^{\bar{c}} \quad \searrow^{c}$$

$$\boxed{xxxx} \qquad \boxed{xxxx}$$

taut taut

tautology, therefore
x1x1 not relatively
essential

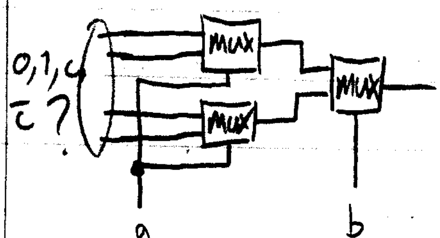
5 3)d)

$$\begin{pmatrix} \cancel{xx00} \\ 0xxx \\ x1x1 \\ \cancel{1x0x} \end{pmatrix}_{xx11} = \begin{matrix} 0xxx \\ x1xx \end{matrix}$$

unate
no tautological
cubes,
not a tautology,
is relatively
essential

③ 3)e) For each cube in the cover that is not relatively essential, check to see whether the cube is covered by the relatively essential cubes. If so, it is totally redundant, if not, it is not totally redundant.

⑤ 4) a)



② b) 1 MUX is required, $M(2)=1$

⑦ c) $M(n) = 2 \cdot M(n-1) + 1$

⑥ d) $M(n) \approx 2^n$

$M(2)=1$
$M(3)=3$
$M(4)=7$
$M(5)=15$
$M(6)=31$

optional:

$M(n) = 2^{(n-1)} - 1$

test:

$$M(n) = 2 \cdot M(n-1) + 1$$
$$2^{(n-1)} - 1 = 2\left(2^{(n-2)} - 1\right) + 1$$
$$\cancel{2^{(n-1)}} - 1 = \cancel{2^{(n-1)}} - 2 + 1$$
$$-1 = -2 + 1$$
$$-1 = -1 \quad \text{o.k.}$$

2 5)a)

a	b	j	k
0	0	1	1
0	1	0	1
1	0	0	1
1	1	0	0

$j(a,b) = \overline{a+b} = \bar{a} \cdot \bar{b}$

$k(a,b) = \overline{ab} = \bar{a} + \bar{b}$

2 5)b)

j	k	f
0	0	0
0	1	1
1	0	1
1	1	0

$f(j,k) = \bar{j}k + j\bar{k}$

2 5)c) $f = (\overline{\bar{a} \cdot \bar{b}}) \cdot (\bar{a} + \bar{b}) + \bar{a}\bar{b} \cdot \overline{(\bar{a}+\bar{b})}$

$f = (a+b)(\bar{a}+\bar{b}) + \bar{a}\bar{b} \cdot ab$

8 literals $\bar{a}\bar{b}ab = 0$ so can claim 4 lit.

2 5)d) Output ignored for certain inputs.

2 5)e) Input can never occur.

③ 5)f) G's truth table doesn't change.

$j=1, k=0$ never happens.

j	k	f
0	0	0
0	1	1
1	0	X
1	1	0

Sat. DC

② 5)g) j, k same

$f(j,k) = \bar{j}k$

③ 5)h) $f = \overline{\bar{a} \cdot \bar{b}} \cdot (\bar{a} + \bar{b})$

$f = (a+b)(\bar{a}+\bar{b})$

4 literals

② 5)i)

to help get f

a	b	j	k	f
0	0	1	1	0
0	1	0	1	1
1	0	0	1	1
1	1	0	0	0

f	a = 0	a = 1
b = 0	0	1
b = 1	1	0

$f = a\bar{b} + \bar{a}b$

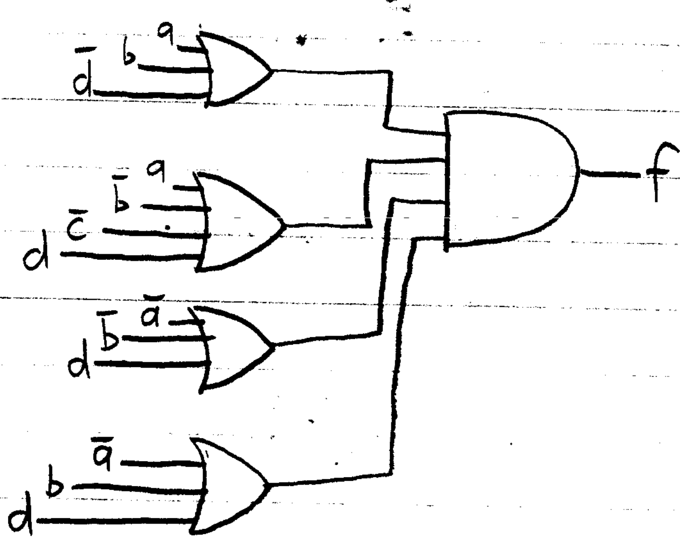
4 literals

(4) 6)a)

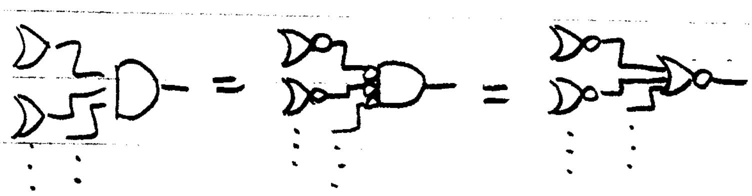
cd \ ab				
	1	1	X	0
	X	X	X	1
	0	1	0	1
	1	0	1	0

$$f = (a+b+\bar{d})(a+\bar{b}+\bar{c}+d)(\bar{a}+\bar{b}+\bar{d})(\bar{a}+b+d)$$

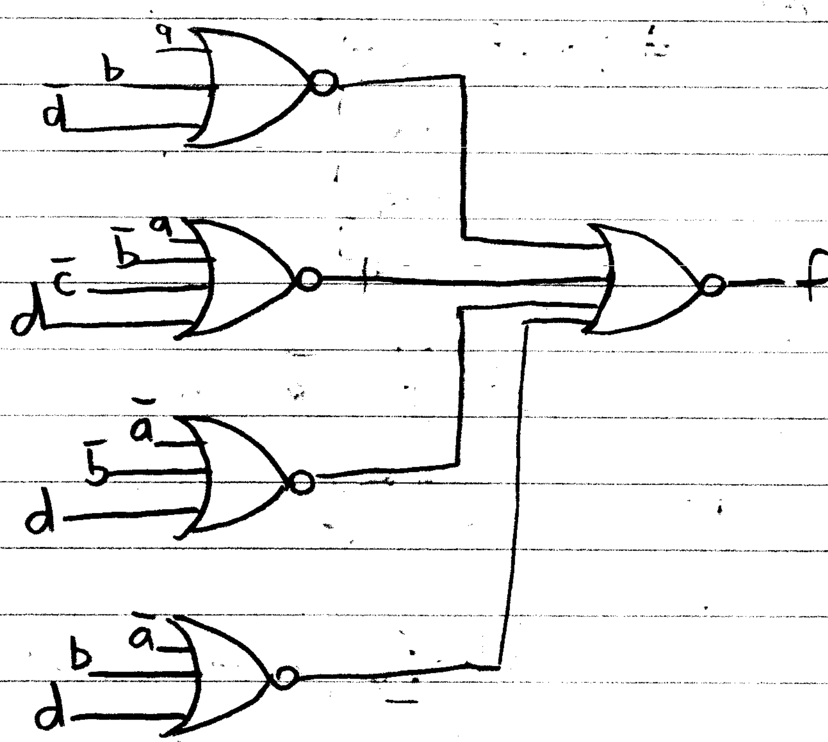
(4) 6)b)



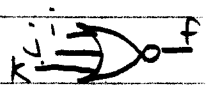
(5) 6)c)



6)c)



⑦ 6)d)



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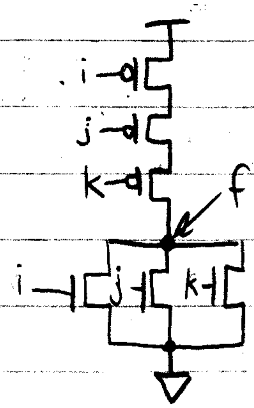
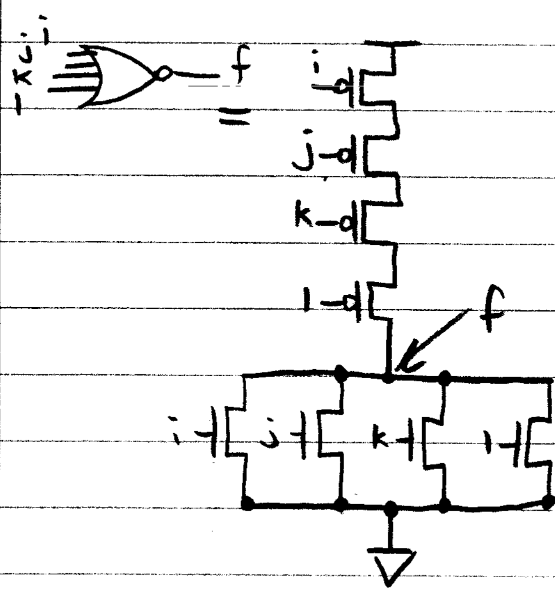


6)d)



Alternative (will cover in class):