Advanced Digital Logic Design – EECS 303

http://ziyang.eecs.northwestern.edu/eecs303/

Teacher:	Robert Dick
Office:	L477 Tech
Email:	dickrp@northwestern.edu
Phone:	847–467–2298

NORTHWESTERN
UNIVERSITY

Outline

Unix privacy hint

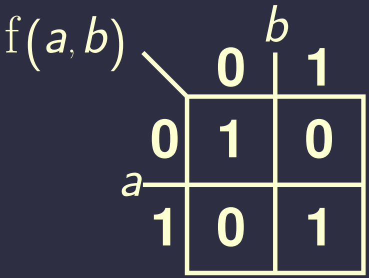
```
chmod -R go-rwx ~
```

Letter	Meaning	Letter	Meaning
u	user	r	read
g	group	w	write
o	other	x	execute

Outline

Section outline

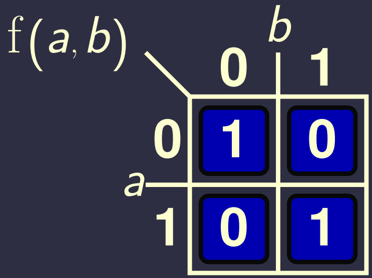
Two-level logic is necessary



Some Boolean functions can not be represented with one logic level

$$(\overline{a}\ \overline{b}) + (a\ b)$$

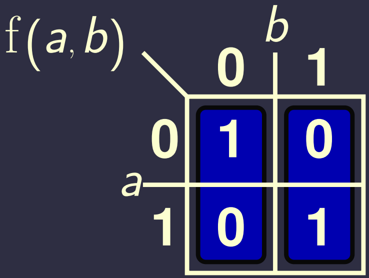
Two-level logic is necessary



Some Boolean functions can not be represented with one logic level

$$(\overline{a}\ \overline{b}) + (a\ b)$$

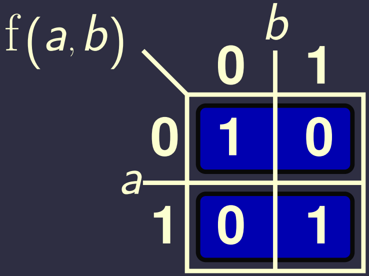
Two-level logic is necessary



Some Boolean functions can not be represented with one logic level

$$(\bar{a}\ \bar{b}) + (a\ b)$$

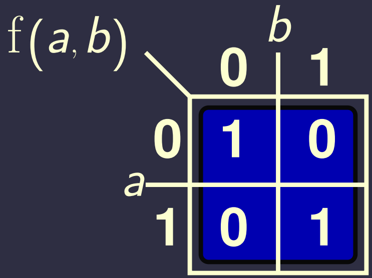
Two-level logic is necessary



Some Boolean functions can not be represented with one logic level

$$(\bar{a}\ \bar{b}) + (a\ b)$$

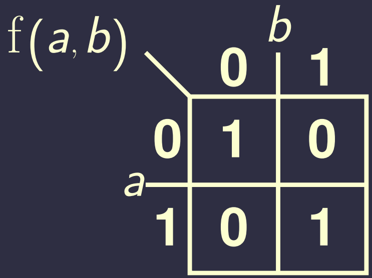
Two-level logic is necessary



Some Boolean functions can not be represented with one logic level

$$(\overline{a}\ \overline{b}) + (a\ b)$$

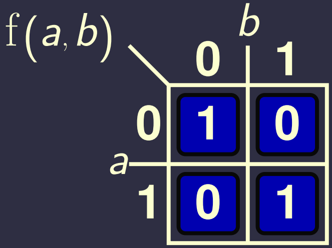
Two-level logic is necessary



Some Boolean functions can not be represented with one logic level

$$(\overline{a}\ \overline{b}) + (a\ b)$$

Two-level logic is sufficient



- All Boolean functions can be represented with two logic levels
- Given k variables, 2^K minterm functions exist
- Select arbitrary union of minterms

Two-level well-understood

- As we will see later, optimal minimization techniques known for two-level
- However, optimal two-level solution may not be optimal solution
  - Sometimes a suboptimal solution to the right problem is better than the optimal solution to the wrong problem

Two-level sometimes impractical

$f(a,b,c,d)$ ab \ cd	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

Two-level sometimes impractical

$f(a,b,c,d)$

ab \ cd	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

Consider a 4-term XOR (parity) gate: $a \oplus b \oplus c \oplus d$

$(\overline{a}\ \overline{b}\ \overline{c}\, d) + (\overline{a}\ \overline{b}\ c\ \overline{d}) + (\overline{a}\ b\ \overline{c}\ \overline{d}) + (\overline{a}\ b\ c\ d) + (a\ b\ \overline{c}\ d) +$
$(a\ b\ c\ \overline{d}) + (a\ \overline{b}\ \overline{c}\ \overline{d}) + (a\ \overline{b}\ c\ d)$

Two-level weakness

- Two-level representation is exponential
- However, it's a simple concept
  - Is $\sum_i^n x_i$ odd?
- Problem with representation, not function

Two-level weakness

Two-level representations also have other weaknesses

- Conversion from SOP to POS is difficult
  - Inverting functions is difficult
  - $\cdot$-ing two SOPs or $+$ing two POSs is difficult
- Neither general POS or SOP are canonical
  - Equivalence checking difficult
- POS satisfiability $\in \mathcal{NP}$-complete

Logic minimization motivation

- Want to reduce area, power consumption, delay of circuits
- Hard to exactly predict circuit area from equations
- Can approximate area with SOP cubes
- Minimize number of cubes and literals in each cube
- Algebraic simplification difficult
  - Hard to guarantee optimality

Logic minimization motivation

- K-maps work well for small problems
  - Too error-prone for large problems
  - Don't ensure optimal prime implicant selection
- Quine–McCluskey optimal and can be run by a computer
  - Too slow on large problems
- Espresso heuristic usually gets good results fast on large problems

Section outline

2. Two-level logic
   - Two-level logic properties
   - Two-level minimization
   - The Quine–McCluskey algorithm

Review: Algebraic simplification

Prove $XY + X\overline{Y} = X$

$XY + X\overline{Y} = X(Y + \overline{Y})$	distributive law
$X(Y + \overline{Y}) = X(1)$	complementary law
$X(1) = X$	identity law

Boolean function minimization

- Algebraic simplification
  - Not systematic
  - How do you know when optimal solution has been reached?
- Optimal algorithm, e.g., Quine–McCluskey
  - Only fast enough for small problems
  - Understanding these is foundation for understanding more advanced methods
- Not necessarily optimal heuristics
  - Fast enough to handle large problems

Section outline

2. Two-level logic
   - Two-level logic properties
   - Two-level minimization
   - The Quine–McCluskey algorithm

Quine–McCluskey two-level logic minimization

- Compute prime implicants with a well-defined algorithm
  - Start from minterms
  - Merge adjacent implicants until further merging impossible
- Select minimal cover from prime implicants
  - Unate covering problem

Computing prime implicants

$\sum = 0$	**0000**
$\sum = 1$	**0001** **0010** **1000**
$\sum = 2$	**1001** **1010**
$\sum = 3$	**1101** **1110**
$\sum = 4$	**1111**

Computing prime implicants

$\sum = 0$	0000	000X
$\sum = 1$	0001 0010 1000	
$\sum = 2$	1001 1010	
$\sum = 3$	1101 1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
$\sum = 1$	0001 0010 1000	
$\sum = 2$	1001 1010	
$\sum = 3$	1101 1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X 00X0
$\sum = 1$	0001 0010 1000	
$\sum = 2$	1001 1010	
$\sum = 3$	1101 1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X 00X0
$\sum = 1$	0001 0010 1000	
$\sum = 2$	1001 1010	
$\sum = 3$	1101 1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000
$\sum = 1$	0001 0010 1000	
$\sum = 2$	1001 1010	
$\sum = 3$	1101 1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000
$\sum = 1$	0001 0010 1000	
$\sum = 2$	1001 1010	
$\sum = 3$	1101 1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000
$\sum = 1$	0001 0010 1000	X001
$\sum = 2$	1001 1010	
$\sum = 3$	1101 1110	
$\sum = 4$	1111	

Computing prime implicants

$\Sigma = 0$	**0000**	**000X**
		00X0
		X000
$\Sigma = 1$	**0001**	**X001**
	0010	
	1000	
$\Sigma = 2$	**1001**	
	1010	
$\Sigma = 3$	**1101**	
	1110	
$\Sigma = 4$	**1111**	

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000
$\sum = 1$	0001 0010 1000	X001 X010
$\sum = 2$	1001 1010	
$\sum = 3$	1101 1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	
$\sum = 2$	1001	
	1010	
$\sum = 3$	1101	
	1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000
$\sum = 1$	0001 0010 1000	X001 X010 100X
$\sum = 2$	1001 1010	
$\sum = 3$	1101 1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000
$\sum = 1$	0001 0010 1000	X001 X010 100X
$\sum = 2$	1001 1010	
$\sum = 3$	1101 1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\sum = 2$	1001	
	1010	
$\sum = 3$	1101	
	1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\sum = 2$	1001	
	1010	
$\sum = 3$	1101	
	1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\sum = 2$	1001	1X01
	1010	
$\sum = 3$	1101	
	1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\sum = 2$	1001	1X01
	1010	
$\sum = 3$	1101	
	1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\sum = 2$	1001	1X01
	1010	1X10
$\sum = 3$	1101	
	1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\sum = 2$	1001	1X01
	1010	1X10
$\sum = 3$	1101	
	1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\sum = 2$	1001	1X01
	1010	1X10
$\sum = 3$	1101	111X
	1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\sum = 2$	1001	1X01
	1010	1X10
$\sum = 3$	1101	111X
	1110	
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\sum = 2$	1001	1X01
	1010	1X10
$\sum = 3$	1101	111X
	1110	11X1
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X
		00X0
		X000
$\sum = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\sum = 2$	1001	1X01
	1010	1X10
$\sum = 3$	1101	111X
	1110	11X1
$\sum = 4$	1111	

Computing prime implicants

$\sum = 0$	0000	000X	X00X
		00X0	
		X000	
$\sum = 1$	0001	X001	
	0010	X010	
	1000	100X	
		10X0	
$\sum = 2$	1001	1X01	
	1010	1X10	
$\sum = 3$	1101	111X	
	1110	11X1	
$\sum = 4$	1111		

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000	X00X
$\sum = 1$	0001 0010 1000	X001 X010 100X 10X0	
$\sum = 2$	1001 1010	1X01 1X10	
$\sum = 3$	1101 1110	111X 11X1	
$\sum = 4$	1111		

Computing prime implicants

$\sum = 0$	0000	000X	X00X
		00X0	X0X0
		X000	
$\sum = 1$	0001	X001	
	0010	X010	
	1000	100X	
		10X0	
$\sum = 2$	1001	1X01	
	1010	1X10	
$\sum = 3$	1101	111X	
	1110	11X1	
$\sum = 4$	1111		

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000	X00X X0X0
$\sum = 1$	0001 0010 1000	X001 X010 100X 10X0	
$\sum = 2$	1001 1010	1X01 1X10	
$\sum = 3$	1101 1110	111X 11X1	
$\sum = 4$	1111		

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000	X00X X0X0
$\sum = 1$	0001 0010 1000	X001 X010 100X 10X0	
$\sum = 2$	1001 1010	1X01 1X10	
$\sum = 3$	1101 1110	111X 11X1	
$\sum = 4$	1111		

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000	X00X X0X0
$\sum = 1$	0001 0010 1000	X001 X010 100X 10X0	
$\sum = 2$	1001 1010	1X01 1X10	
$\sum = 3$	1101 1110	111X 11X1	
$\sum = 4$	1111		

Computing prime implicants

$\sum = 0$	0000	000X 00X0 X000	X00X X0X0
$\sum = 1$	0001 0010 1000	X001 X010 100X 10X0	
$\sum = 2$	1001 1010	1X01 1X10	
$\sum = 3$	1101 1110	111X 11X1	
$\sum = 4$	1111		

Computing prime implicants

$\sum = 0$	0000	000X	X00X
		00X0	X0X0
		X000	
$\sum = 1$	0001	X001	
	0010	X010	
	1000	100X	
		10X0	
$\sum = 2$	1001	1X01	
	1010	1X10	
$\sum = 3$	1101	111X	
	1110	11X1	
$\sum = 4$	1111		

Definition: Unate covering

Given a matrix for which all entries are 0 or 1, find the minimum cardinality subset of columns such that, for every row, at least one column in the subset contains a 1.

Definition: Unate covering

Given a matrix for which all entries are 0 or 1, find the minimum cardinality subset of columns such that, for every row, at least one column in the subset contains a 1.

I'll give an example

Prime implicant selection

	01X	0X0	X00	X11
000		1	1	
010	1	1		
011	1			1
111				1
100			1	

Prime implicant selection

	01X	0X0	X00	X11
~~000~~		1	1	
010	1	1		
011	1			1
111				1
~~100~~			1	

Prime implicant selection

	01X	0X0	X00	X11
~~000~~		1	1	
010	1	1		
~~011~~	1			1
~~111~~				1
~~100~~			1	

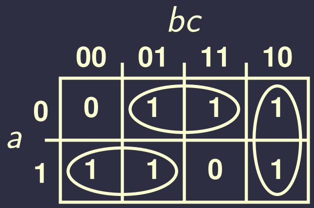
Prime implicant selection

	01X	0X0	X00	X11
~~000~~		1	1	
~~010~~	1	1		
~~011~~	1			1
~~111~~				1
~~100~~			1	

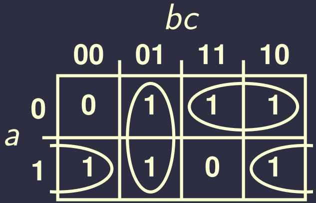
Cyclic core

a \ bc	00	01	11	10
0	0	1	1	1
1	1	1	0	1

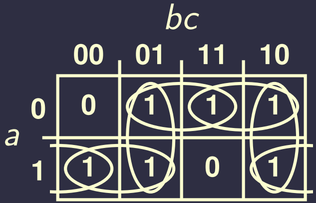
Cyclic core



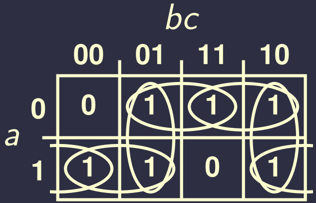
Cyclic core



Cyclic core



Cyclic core



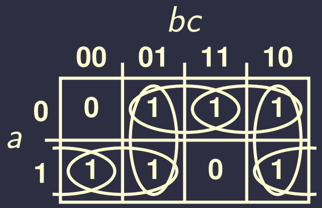
	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Cyclic core

a \ bc	00	01	11	10
0	0	1	1	1
1	1	1	0	1

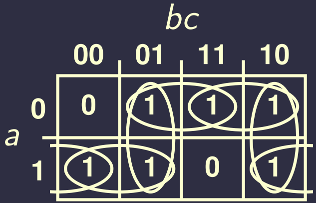
	0X1	01X	X01	X10	10X	1X0
~~001~~	1		1			
~~011~~	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Cyclic core



	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Cyclic core



	0X1	01X	X01	X10	10X	1X0
001	1		1			
~~011~~	1	1				
~~010~~		1		1		
100					1	1
101			1		1	
110				1		1

Cyclic core

a \ bc	00	01	11	10
0	0	1	1	1
1	1	1	0	1

	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Implicant selection reduction

- Eliminate rows covered by essential columns
- Eliminate rows dominated by other rows
- Eliminate columns dominated by other columns

Eliminate rows covered by essential columns

	A	B	C
H		1	
I	1		1
J	1	1	
K		1	1

Eliminate rows covered by essential columns

	A	B	C
H		1	
I	1		1
J	1	1	
K		1	1

Eliminate rows covered by essential columns

	A	B	C
H		1	
I	1		1
J	1	1	
K		1	1

Eliminate rows covered by essential columns

	A	B	C
H		1	
I	1		1
J	1	1	
K		1	1

Eliminate rows dominating other rows

	A	B	C
H	1		
I	1	1	
J	1		1

Eliminate rows dominating other rows

	A	B	C
H	1		
I	1	1	
J	1		1

Eliminate rows dominating other rows

	A	B	C
H	1		
I	1	1	
J	1		1

Eliminate columns dominated by other columns

	A	B	C
H	1		
I	1	1	
J	1		1
K		1	

Eliminate columns dominated by other columns

	A	B	C
H	1		
I	1	1	
J	1		1
K		1	

Eliminate columns dominated by other columns

	A	B	C
H	1		
I	1	1	
J	1		1
K		1	

Backtracking

- Will proceed to complete solution unless cyclic
- If cyclic, can bound cover size
    - Compute independent sets

Find lower bound

a \ bc	00	01	11	10
0	0	1	1	1
1	1	1	0	1

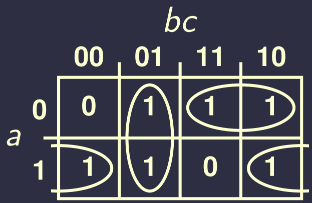
	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Find lower bound

a \ bc	00	01	11	10
0	0	1	1	1
1	1	1	0	1

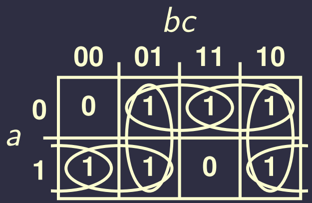
	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Find lower bound



	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Find lower bound



	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

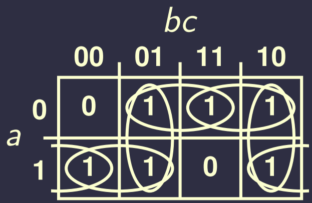
Find lower bound

bc

a	00	01	11	10
0	0	1	1	1
1	1	1	0	1

	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

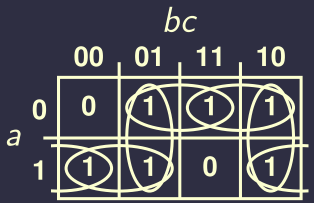
Find lower bound



	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

3 disjoint rows → 3 columns required

Find lower bound



	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

3 disjoint rows $\longrightarrow$ 3 columns required

Use bound to constrain search space

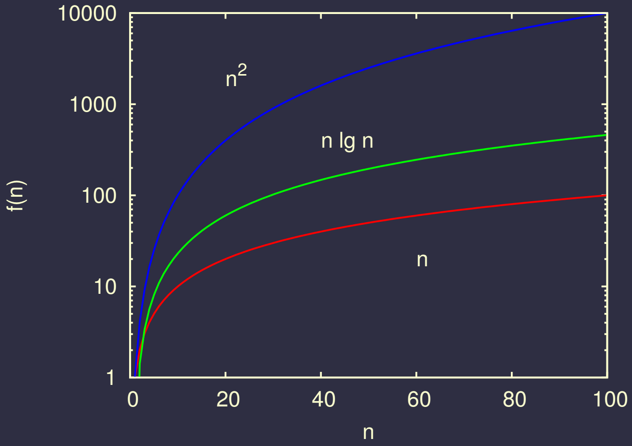
- Eliminate rows covered by essential columns
- Eliminate rows dominated by other rows
- Eliminate columns dominated by other columns
- Branch-and-bound on cyclic problems
  - Use independent sets to bound
- Speed improved, still $\in \mathcal{NP}$-complete

Outline

Section outline

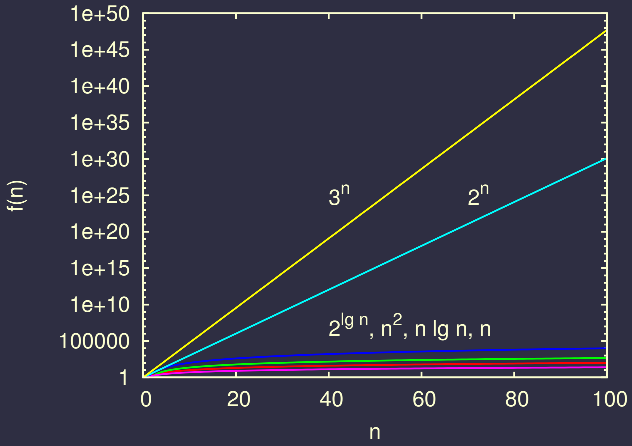
Extremely brief introduction to $\mathcal{NP}$-completeness

Polynomial-time algorithms: $\mathcal{O}(n)$, $\mathcal{O}(n \lg n)$, $\mathcal{O}(n^2)$



Extremely brief introduction to $\mathcal{NP}$-completeness

There also exist exponential-time algorithms: $\mathcal{O}\left(2^{\lg n}\right)$, $\mathcal{O}\left(2^n\right)$, $\mathcal{O}\left(3^n\right)$



Extremely brief introduction to $\mathcal{NP}$-completeness

- Any $\mathcal{NP}$-complete problem instance can be converted to any other $\mathcal{NP}$-complete problem instance in polynomial time (quickly)
- Nobody has ever developed a polynomial time (fast) algorithm that optimally solves an $\mathcal{NP}$-complete problem
- It is generally believed (but not proven) that it is not possible to devise a polynomial time (fast) algorithm that optimally solves an $\mathcal{NP}$-complete problem
- Can use heuristics

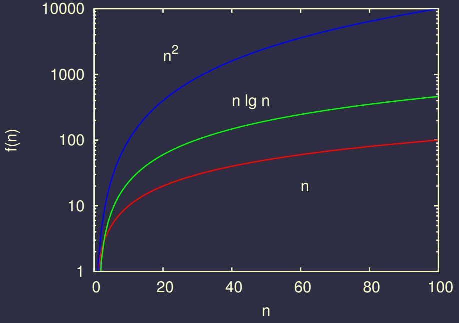
Extremely brief introduction to $\mathcal{NP}$-completeness

- Any $\mathcal{NP}$-complete problem instance can be converted to any other $\mathcal{NP}$-complete problem instance in polynomial time (quickly)
- Nobody has ever developed a polynomial time (fast) algorithm that optimally solves an $\mathcal{NP}$-complete problem
- It is generally believed (but not proven) that it is not possible to devise a polynomial time (fast) algorithm that optimally solves an $\mathcal{NP}$-complete problem
- Can use heuristics
  - Fast algorithms that often produce good solutions

$\mathcal{NP}$-completeness

Recall that sorting may be done in $\mathcal{O}(n \lg n)$ time

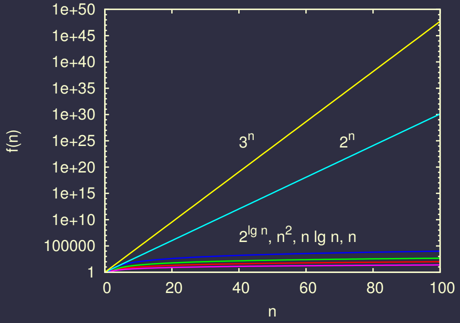
DFS $\in \mathcal{O}(|V| + |E|)$, BFS $\in \mathcal{O}(|V|)$, Topological sort $\in$



$\mathcal{O}(|V| + |E|)$

$\mathcal{NP}$-completeness

There also exist exponential-time algorithms: $\mathcal{O}\left(2^{\lg n}\right)$, $\mathcal{O}\left(2^n\right)$, $\mathcal{O}\left(3^n\right)$



$\mathcal{NP}$-completeness

For $t(n) = 2^n$ seconds

$$
\begin{aligned}
t(1) &= 2 \text{ seconds} \\
t(10) &= 17 \text{ minutes} \\
t(20) &= 12 \text{ days} \\
t(50) &= 35,702,052 \text{ years} \\
t(100) &= 40,196,936,841,331,500,000,000 \text{ years}
\end{aligned}
$$

$\mathcal{NP}$-completeness

- Digital design and synthesis is full of NP-complete problems
- Graph coloring
- Scheduling
- Graph partitioning
- Satisfiability (and 3SAT)
- *Covering*
- ...and many more

$\mathcal{NP}$-completeness

- There is a class of problems, $\mathcal{NP}$-complete, for which nobody has found polynomial time solutions
- It is possible to convert between these problems in polynomial time
- Thus, if it is possible to solve any problem in $\mathcal{NP}$-complete in polynomial time, all can be solved in polynomial time
- Unproven conjecture: $\mathcal{NP} \neq \mathcal{P}$

$\mathcal{NP}$-completeness

- What is $\mathcal{NP}$? Nondeterministic polynomial time.
- A computer that can simultaneously follow multiple paths in a solution space exploration tree is nondeterministic. Such a computer can solve $\mathcal{NP}$ problems in polynomial time.
- I.e., a computer that can simultaneously be in multiple states.
- Nobody has been able to prove either

$$\mathcal{P} \neq \mathcal{NP}$$

or

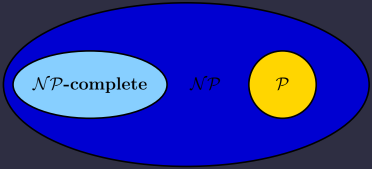
$$\mathcal{P} = \mathcal{NP}$$

$\mathcal{NP}$-completeness

If we define $\mathcal{NP}$-complete to be a set of problems in $\mathcal{NP}$ for which any problem's instance may be converted to an instance of another problem in $\mathcal{NP}$-complete in polynomial time, then

$$\mathcal{P} \subsetneq \mathcal{NP} \Rightarrow \mathcal{NP}\text{-complete} \cap \mathcal{P} = \varnothing$$

Basic complexity classes



- $\mathcal{P}$ solvable in polynomial time by a computer (Turing Machine)
- $\mathcal{NP}$ solvable in polynomial time by a nondeterministic computer
- $\mathcal{NP}$-complete converted to other $\mathcal{NP}$-complete problems in polynomial time

Section outline

How to deal with hard problems

- What should you do when you encounter an apparently hard problem?
- Is it in $\mathcal{NP}$-complete?
- If not, solve it
- If so, then what?

How to deal with hard problems

- What should you do when you encounter an apparently hard problem?
- Is it in $\mathcal{NP}$-complete?
- If not, solve it
- If so, then what?

Despair.

How to deal with hard problems

- What should you do when you encounter an apparently hard problem?
- Is it in $\mathcal{NP}$-complete?
- If not, solve it
- If so, then what?

Solve it!

How to deal with hard problems

- What should you do when you encounter an apparently hard problem?
- Is it in $\mathcal{NP}$-complete?
- If not, solve it
- If so, then what?

Resort to a suboptimal heuristic.
Bad, but sometimes the only choice.

How to deal with hard problems

- What should you do when you encounter an apparently hard problem?
- Is it in $\mathcal{NP}$-complete?
- If not, solve it
- If so, then what?

Develop an approximation algorithm.
Better.

How to deal with hard problems

- What should you do when you encounter an apparently hard problem?
- Is it in $\mathcal{NP}$-complete?
- If not, solve it
- If so, then what?

Determine whether all encountered problem instances are constrained.
Wonderful when it works.

One example

O. Coudert. Exact coloring of real-life graphs is easy. *Design Automation*, pages 121–126, June 1997.

Outline

Section outline

4. Espresso
   Heristic logic minimization

Heuristic logic minimization

Optimal two-level logic synthesis is $\mathcal{NP}$-complete

- Upper bound on number of prime implicants grows $^{3^n}/_n$ where n is the number of inputs
- Given > 16 inputs, can be intractable
- However, there have been advances in complete solvers for many functions
  - Optimal solutions are possible for some large functions

Heuristic logic minimization

- For difficult and large functions, solve by heuristic search
- Multi-level logic minimization is also best solved by search
- The general search problem can be introduced via two-level minimization
  - Examine simplified version of the algorithms in Espresso

Espresso two-level logic minimization heuristic

- Generate only a subset of prime implicants
- Carefully selects subset of prime implicants covering on-set
- Guaranteed to be correct
  - May not be optimal

Espresso

Can be viewed in the following

- Start with a potentially optimal algorithm
- Add numerous techniques for constraining the search space
- Uses efficient move order to allow pruning
- Disable backtracking to arrive at a heuristic solver
- Widely used in industry
- Still has room for improvement
  - E.g., early recursion termination

Summary

- Properties of two-level logic
- The Quine-McCluskey (tabular) method
- $\mathcal{NP}$-complete: Why use heuristics?
- Espresso

Outline

1. Misc.
2. Two-level logic
3. Computational complexity
4. Espresso
5. Homework

Homework assignment one

- Algebraic manipulation (Review)
- K-Maps (Review)
- Quine-McCluskey
- Espresso

Next lecture

- More on Espresso algorithm
- Technologies and implementation methods

Reading assignment

- http://www.writphotec.com/mano/reading_supplements.html
- *More Optimization* for Quine–McCluskey